

# Freezing of Moduli by N=2 Dyons

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## Abstract

In N=2 ungauged supergravity we have found the most general double-extreme dyonic black holes with arbitrary number  $n_v$  of constant vector multiplets and  $n_h$  of constant hypermultiplets. They are double-extreme: 1) supersymmetric with coinciding horizons, 2) the mass for a given set of quantized charges is extremal. The spacetime is of the Reissner-Nordström form and the vector multiplet moduli depend on dyon charges.

As an example we display  $n_v$  complex moduli as functions of  $2(n_v + 1)$  electric and magnetic charges in a model related to a classical Calabi-Yau moduli space. A specific case includes the complex S, T, U moduli depending on 4 electric and 4 magnetic charges of 4 U(1) gauge groups.

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# I. INTRODUCTION

Supersymmetric black holes in most general version of ungauged N=2 supergravity interacting with arbitrary number  $n_v$  of vector multiplets and  $n_h$  of hypermultiplets have various properties which can be studied in a relatively easy way. This happens due to the existence of the well developed theory of local N=2 supersymmetry based on special and quaternionic geometry, see for example [1,2] and references therein.

There is a long standing problem to find explicitly the most general family of supersymmetric black holes. In this paper we will find all supersymmetric black holes solutions of N=2 theory for which moduli are frozen and do not change their values all the way from the horizon to infinity. As we will see these are highly non-trivial solutions since each of the  $(n_v + 1)$  U(1) gauge group will be allowed to carry arbitrary electric and magnetic charges  $(p^\Lambda, q_\Lambda)$ ,  $\Lambda = 0, \dots, n_v$ , which will fix the complex  $n_v$  moduli  $(z^i, \bar{z}^i)$ ,  $i = 1, \dots, n_v$  of vector multiplets.

The most important properties of generic supersymmetric regular black holes with non-constant moduli in N=2 theory have been established recently: moduli of vector multiplets near the horizon become functions of charges only [3]. In fact it has been found that near the horizon supersymmetric black holes choose the size of the Bertotti-Robinson throat to be related to the extremal value of the central charge [4].

$$M_{\text{BR}}^2(p, q) = \left(|Z|^2\right)_{\frac{\partial|Z(z, \bar{z}, (p, q))|}{\partial z}=0} \quad . \quad (1)$$

In this paper we will introduce the concept of a double-extreme black hole. The term “non-extreme black hole” is usually applied to a charged black hole which in general has two horizons. When they coincide the black hole is called “extreme”. Typically extreme black holes have some unbroken supersymmetry. When they solve equations of motion of supergravities the mass of the extreme black hole depends on moduli as well as on quantized charges. When the mass is extremized in the moduli space at fixed charges, moduli become functions of charges. We define *double-extreme black holes as extreme, supersymmetric black holes with the extremal value of the ADM mass equal to the Bertotti-Robinson mass.*

$$M_{\text{ADM}}^2 = M_{\text{BR}}^2(p, q) . \quad (2)$$

Double-extreme black holes have constant moduli both for vector multiplets as well as for hypermultiplets but unconstrained electric and magnetic charges  $(p^\Lambda, q_\Lambda)$  in each of  $n_v + 1$  gauge group. The attractor diagram in Fig.1 of ref. [4] shows the double-extreme black holes on the horizontal line with the zero slope.

The relation between charges and moduli for supersymmetric black holes near the horizon was established in the following form [4]

$$\begin{pmatrix} p^\Lambda \\ q_\Lambda \end{pmatrix} = \text{Re} \begin{pmatrix} 2i\bar{Z}L^\Lambda \\ 2i\bar{Z}M_\Lambda \end{pmatrix} , \quad (3)$$

where  $Z$  is the central charge depending on moduli and on conserved charges  $(p^\Lambda, q_\Lambda)$  and  $(L^\Lambda, M_\Lambda)$  are covariantly holomorphic sections depending on moduli. This is a highly non-trivial constraint between moduli and charges. Only few solutions to this constraint are known [4].

In this paper we will show how to solve this constraint and find the double-extreme black holes moduli (or the near horizon values of moduli of extreme black holes with non-constant moduli) in case of two models. The first one has the prepotential  $F = -iX^0X^1$  and there is also a symplectic transformation of this model to the one without the prepotential. The second one is the model known as ST[2,n] manifold and has no prepotential. Upon symplectic transformation this model is related to some classical Calabi-Yau moduli space described by the prepotential of the form  $F = d_{ABC}\frac{X^AX^BX^C}{X^0}$ . We will find the fixed values of  $n_v = (n+1)$  complex moduli as the function of  $2(n_v+1) = 2(n+2)$  electric and magnetic charges in this model. In the race for most general black hole solutions this seems at the moment to be the most general available case of relations between charges, space-time geometry and moduli. We will find out also that the expressions for the moduli in terms of charges are surprisingly elegant. Our example for the ST[2,2] case is related via symplectic transformation to the so-called STU model. Thus we will get the fixed values of these 3 complex moduli as the functions of 4 electric and 4 magnetic charges.

The paper is organized as follows. In Section 2 we look for the solution of field equations for the double-extreme black holes in theories with arbitrary prepotentials or symplectic sections. The most important part of this which differs vastly from the standard routine of solving field equations for black holes is the procedure of solving equations for constant moduli. It is this place where the properties of special geometry and holomorphic properties are of crucial importance and supply the solutions. Section 3 presents double-extreme black holes and frozen moduli as function of charges for the theory with the prepotential  $F = -iX^0X^1$  describing N=2 supergravity interacting with one vector multiplet corresponding to SO(4) version of N=4 supergravity. We also get the black holes and frozen moduli for the symplectic transformation of this model which is related to SU(4) version of N=4 supergravity and as N=2 theory has no prepotential. Finally Section 4 deals with ST[2,n] manifold which is an  $\frac{SU(1,1)}{U(1)} \times \frac{SO(2,n)}{SO(2) \times SO(n)}$  symmetric manifold. This N=2 theory has no prepotential. This theory is related via symplectic transformation to the one with the cubic holomorphic prepotential associated with particular Calabi-Yau moduli space. We will solve the moduli stabilization equations (3) and express vector multiplet moduli in terms of double-extreme dyon charges  $(p, q)$ . In Discussion we summarize the new results and speculate about the possibility to construct global SUSY theories with stabilization of moduli due to the presence of F-I terms or hypermultiplet charges in the action.

## II. N=2 DOUBLE-EXTREME BLACK HOLES

We will present here the minimal information on N=2 ungauged supergravity which will be necessary to explain the action which is our starting point for the derivation of N=2 black holes.

Ungauged d=4 , N=2 supergravity theory includes the following multiplets:

1. **gravitational multiplet** contains the vielbein, the SU(2) doublet of gravitino and the graviphoton;
2.  $n_v$  **vector multiplets**, each contains a gauge boson, a doublet of gauginos and a com-

plex scalar field  $z$ ;

3.  $n_h$  **hypermultiplets**, each contains a doublet of hyperinos and 4 real scalar fields  $q$ .

Complex scalar fields  $z^i$  ( $i = 1, 2, \dots, n_v$ ) of N=2 vector multiplets can be regarded as coordinates of a special Kähler manifold of dimension  $n_v$  with additional constraint on curvature. Scalar fields  $q^u$  ( $u = 1, \dots, 4n_h$ ) of  $n_h$  hypermultiplets can be considered as  $4n_h$  coordinates of a quaternionic manifold, which is a  $4n_h$ -dimensional real manifold with a metric  $h_{uv}(q)$ . Special Kähler manifold can be defined by constructing flat symplectic bundle of dimension  $2n_v + 2$  over Kähler-Hodge manifold with symplectic section defined as

$$V = (L^\Lambda, M_\Lambda), \quad \Lambda = 0, 1, \dots, n_v, \quad (4)$$

where  $(L, M)$  obey the symplectic constraint  $i(\bar{L}^\Lambda M_\Lambda - L^\Lambda \bar{M}_\Lambda) = 1$  and  $L^\Lambda(z, \bar{z})$  and  $M_\Lambda(z, \bar{z})$  depend on scalar fields  $z, \bar{z}$ , which are the coordinates of the “moduli space”.  $L^\Lambda$  and  $M_\Lambda$  are covariantly holomorphic (with respect to Kähler connection), e.g.

$$D_{\bar{k}} L^\Lambda = (\partial_{\bar{k}} - \frac{1}{2} K_{\bar{k}}) L^\Lambda = 0, \quad (5)$$

where  $K$  is the Kähler potential. Symplectic invariant form of the Kähler potential can be found from this equation by introducing the holomorphic section  $(X^\Lambda, F_\Lambda)$ :

$$L^\Lambda = e^{K/2} X^\Lambda, \quad M_\Lambda = e^{K/2} F_\Lambda, \quad (\partial_{\bar{k}} X^\Lambda = \partial_{\bar{k}} F_\Lambda = 0). \quad (6)$$

The Kähler potential is  $K = -\ln i(\bar{X}^\Lambda F_\Lambda - X^\Lambda \bar{F}_\Lambda)$ . The Kähler metric is given by  $g_{k\bar{k}} = \partial_k \partial_{\bar{k}} K$ . Finally from special geometry one finds that there exists a complex symmetric  $(n_v + 1) \times (n_v + 1)$  matrix  $\mathcal{N}_{\Lambda\Sigma}$  such that

$$M_\Lambda = \mathcal{N}_{\Lambda\Sigma} L^\Sigma, \quad \text{Im} \mathcal{N}_{\Lambda\Sigma} L^\Lambda \bar{L}^\Sigma = -\frac{1}{2}, \quad D_{\bar{i}} \bar{M}_\Lambda = \mathcal{N}_{\Lambda\Sigma} D_{\bar{i}} \bar{L}^\Sigma. \quad (7)$$

The bosonic part of ungauged  $N = 2$  supergravity action is given by

$$\begin{aligned} \frac{1}{2} \int d^4x \sqrt{-g} \Big\{ & -R + 2g_{i\bar{j}} \nabla^\mu z^i \nabla_\mu \bar{z}^{\bar{j}} + 2h_{uv} \nabla_\mu q^u \nabla^\mu q^v \\ & + 2(\text{Im} \mathcal{N}_{\Lambda\Sigma} \mathcal{F}^\Lambda \mathcal{F}^\Sigma + \text{Re} \mathcal{N}_{\Lambda\Sigma} \mathcal{F}^{\Lambda*} \mathcal{F}^\Sigma) \Big\}. \end{aligned} \quad (8)$$

The kinetic term of gauge fields is defined by the period matrix  $\mathcal{N}_{\Lambda\Sigma}$  which depends only on scalar fields of the vector multiplets  $z^i$ . The vector field action can be also rewritten as  $(\text{Im}\mathcal{N}_{\Lambda\Sigma}\mathcal{F}^\Lambda\mathcal{F}^\Sigma + \text{Re}\mathcal{N}_{\Lambda\Sigma}\mathcal{F}^{\Lambda*}\mathcal{F}^\Sigma) = \mathcal{F}^{\Lambda*}\mathcal{G}_\Lambda$ , where  $\mathcal{G}_\Lambda = \text{Re}\mathcal{N}_{\Lambda\Sigma}\mathcal{F}^\Sigma - \text{Im}\mathcal{N}_{\Lambda\Sigma}^*\mathcal{F}^\Sigma$ . The symplectic structure of equation of motion is manifest in terms of the  $\text{Sp}(2n_v+2)$  symplectic vector field strength  $(\mathcal{F}^\Lambda, \mathcal{G}_\Lambda)$ . These vector fields in the symplectic basis decompose in the susy basis into the vector field of the gravitational multiplet (graviphoton) and the vector fields of the vector multiplets. The graviphoton is given by the following symplectic invariant combination of the vector fields in the action

$$T = M_\Lambda \mathcal{F}^\Lambda - L^\Lambda \mathcal{G}_\Lambda .$$

The central charge formula (the charge of the graviphoton) for the general N=2 theories is given by [1]

$$Z(z, \bar{z}, q, p) = e^{\frac{K(z, \bar{z})}{2}} (X^\Lambda(z) q_\Lambda - F_\Lambda(z) p^\Lambda) = (L^\Lambda q_\Lambda - M_\Lambda p^\Lambda) . \quad (9)$$

The  $n_v$  vector fields of the vector multiplets are given by the symplectic invariant combination

$$\mathcal{F}^{-i} = g^{i\bar{k}} (D_{\bar{k}} \bar{M}_\Lambda \mathcal{F}^\Lambda - D_{\bar{k}} \bar{L}^\Lambda \mathcal{G}_\Lambda) .$$

The equations of motion for  $\mathcal{F}^*$ , Bianchi identity for  $\mathcal{F}^*$ , equations of motion for  $z^i$ , equations of motion for  $q^u$ , and the Einstein-Maxwell equation are:

$$\nabla_\mu (\text{Im}\mathcal{N}_{\Lambda\Sigma} \mathcal{F}^{\Sigma\mu\nu} + \text{Re}\mathcal{N}_{\Lambda\Sigma}^* \mathcal{F}^{\Sigma\mu\nu}) \equiv \nabla_\mu (*\mathcal{G}_\Lambda^{\mu\nu}) = 0, \quad (10)$$

$$\nabla_\mu (*\mathcal{F}^{\Lambda\mu\nu}) = 0, \quad (11)$$

$$\nabla_\mu (g_{i\bar{k}} \nabla^\mu z^i) - \partial_{\bar{k}} g_{i\bar{j}} \nabla^\mu z^i \nabla_\mu \bar{z}^{\bar{j}} - 2\partial_{\bar{k}} \text{Im}(\mathcal{N}_{\Lambda\Sigma} \mathcal{F}^{+\Lambda} \mathcal{F}^{+\Sigma}) = 0, \quad (12)$$

$$\nabla_\mu (h_{uv} \nabla^\mu q^u) - \partial_w h_{uv} \nabla^\mu q^u \nabla_\mu q^v = 0, \quad (13)$$

$$R_{\mu\nu} + 2g_{i\bar{j}} \nabla_{(\mu} z^i \nabla_{\nu)} \bar{z}^{\bar{j}} + 2h_{uv} \nabla_{(\mu} q^u \nabla_{\nu)} q^v + 4\text{Im}\mathcal{N}_{\Lambda\Sigma} (\mathcal{F}^\Lambda_{\mu\rho} \mathcal{F}^\Sigma_{\nu}{}^\rho - \frac{1}{4} g_{\mu\nu} \mathcal{F}^\Lambda_{\sigma\rho} \mathcal{F}^{\Sigma\sigma\rho}) = 0. \quad (14)$$

Symplectic covariant charges are defined as follows

$$\begin{pmatrix} p^\Lambda \\ q_\Lambda \end{pmatrix} = \begin{pmatrix} \int \mathcal{F}^\Lambda \\ \int \mathcal{G}_\Lambda \end{pmatrix}. \quad (15)$$

We will be looking for double-extreme black holes of N=2 theory with arbitrary charges  $(p, q)$  using the following assumptions. All scalars are constant:

$$\partial_\mu z^i = 0 , \quad \partial_\mu q^u = 0 . \quad (16)$$

Unbroken supersymmetry requires in this case [3,4]

$$\mathcal{F}^{-i} = 0 . \quad (17)$$

We assume the standard form of the metric of supersymmetric black hole with spherical symmetry and asymptotic flatness.

$$ds^2 = e^{2U} dt^2 - e^{-2U} d\vec{x}^2, \quad U = U(r) , \quad U \rightarrow 0 \quad \text{as} \quad r \rightarrow \infty . \quad (18)$$

Our ansatz is

$$\mathcal{F}^\Lambda = e^{2U} \frac{2Q^\Lambda}{r^2} dt \wedge dr - \frac{2P^\Lambda}{r^2} r d\theta \wedge r \sin \theta d\phi , \quad (19)$$

$$e^{-U} = 1 + \frac{M}{r} . \quad (20)$$

We will find that the fields  $\mathcal{F}^\Lambda$  solve Maxwell equation in curved spherical symmetric space-time, with  $e^{-U}$  harmonic and the mass is given in terms of the matrix  $\text{Im}\mathcal{N}$  and the charges of  $\mathcal{F}^\Lambda$ .

$$M^2 = -2 \text{Im}\mathcal{N}_{\Lambda\Sigma} (Q^\Lambda Q^\Sigma + P^\Lambda P^\Sigma) = |Z|^2 . \quad (21)$$

To prove this consider the Maxwell Equation for  $\mathcal{F}^\Lambda$ . First consider (10), the equation of motion for  $\mathcal{F}^\Lambda$ . By (11) and the assumption that the moduli fields are constant, (10) becomes

$$\nabla_\mu (\text{Im}\mathcal{N}_{\Lambda\Sigma} \mathcal{F}^{\Sigma\mu\nu}) = 0 . \quad (22)$$

We can also multiply (11) with  $\text{Im}\mathcal{N}$  to obtain the dual of the above equation

$$\nabla_\mu (\text{Im}\mathcal{N}_{\Lambda\Sigma}^* \mathcal{F}^{\Sigma\mu\nu}) = 0 . \quad (23)$$

Let us note that a field strength  $\mathcal{F}'$  in spherically symmetric spacetime satisfying the Maxwell equations

$$\nabla_\mu(\mathcal{F}'^{\mu\nu}) = 0 , \quad (24)$$

$$\nabla_\mu(*\mathcal{F}'^{\mu\nu}) = 0 , \quad (25)$$

have the following solution

$$\mathcal{F}' = \frac{1}{\sqrt{-g}} \frac{2Q'}{r^2} dt \wedge dr - \frac{2P'}{r^2} r d\theta \wedge r \sin \theta d\phi . \quad (26)$$

Therefore, the solution to (22, 23) is

$$\text{Im} \mathcal{N}_{\Lambda\Sigma} \mathcal{F}^\Sigma = e^{2U} \frac{2Q'^\Lambda}{r^2} dt \wedge dr - \frac{2P'^\Lambda}{r^2} r d\theta \wedge r \sin \theta d\phi . \quad (27)$$

The matrix  $\text{Im} \mathcal{N}$  is negative definite, it can be inverted and

$$\mathcal{F}^\Lambda = e^{2U} \frac{\text{Im} \mathcal{N}_{\Lambda\Sigma}^{-1} 2Q'^\Sigma}{r^2} dt \wedge dr - \frac{\text{Im} \mathcal{N}_{\Lambda\Sigma}^{-1} 2P'^\Sigma}{r^2} r d\theta \wedge r \sin \theta d\phi \quad (28)$$

$$= e^{2U} \frac{2Q^\Lambda}{r^2} dt \wedge dr - \frac{2P^\Lambda}{r^2} r d\theta \wedge r \sin \theta d\phi , \quad (29)$$

which is (19), where  $Q^\Lambda$  and  $P^\Lambda$  are the electric and magnetic charges of  $\mathcal{F}^\Lambda$  respectively.

Equations of motion for the hypermultiplet scalars for constant  $q^u$  is satisfied without any additional restriction. Thus black holes of Abelian theory do not seem to stabilize the coordinates of the quaternionic manifolds. However, equations of motion for vector multiplets moduli (12) are very restrictive when  $z^i$  are constants. We are going to show that it is satisfied with constant moduli fields taking into account that  $\mathcal{F}^{-i} = 0$ . When  $\mathcal{F}^{-i} = 0$ , one can find [1]

$$\begin{pmatrix} \mathcal{F}^{+\Lambda} \\ \mathcal{G}^{+\Lambda} \end{pmatrix} = -ie^{K/2} T^+ \begin{pmatrix} X^\Lambda \\ F_\Lambda \end{pmatrix} . \quad (30)$$

To solve the equation of motion (12) for  $z$  we have to consider

$$\begin{aligned} \partial_{\bar{k}} \text{Im} (\mathcal{N}_{\Lambda\Sigma} \mathcal{F}^{+\Lambda} \mathcal{F}^{+\Sigma}) &= \frac{i}{2} (\partial_{\bar{k}} \bar{\mathcal{N}}_{\Lambda\Sigma} \mathcal{F}^{-\Lambda} \mathcal{F}^{-\Sigma} - \partial_{\bar{k}} \mathcal{N}_{\Lambda\Sigma} \mathcal{F}^{+\Lambda} \mathcal{F}^{+\Sigma}) \\ &= \frac{i}{2} \partial_{\bar{k}} \bar{\mathcal{N}}_{\Lambda\Sigma} \mathcal{F}^{-\Lambda} \mathcal{F}^{-\Sigma} \end{aligned} \quad (31)$$



and take into account that  $\mathcal{N}_{\Lambda\Sigma}$  is holomorphic ( $F_\Lambda = \mathcal{N}_{\Lambda\Sigma} X^\Sigma$ ). Consider the complex conjugate of it, ignoring constant factors

$$\begin{aligned}
\partial_k \mathcal{N}_{\Sigma\Lambda} \mathcal{F}^{+\Lambda} \mathcal{F}^{+\Sigma} &= -\partial_k \mathcal{N}_{\Sigma\Lambda} X^\Lambda X^\Sigma e^K T^{+2} \\
&= -\left[ \partial_k (\mathcal{N}_{\Lambda\Sigma} X^\Lambda) X^\Sigma - \mathcal{N}_{\Lambda\Sigma} \partial_k X^\Lambda X^\Sigma \right] e^K T^{+2} \\
&= -(\partial_k F_\Sigma X^\Sigma - F_\Lambda \partial_k X^\Lambda) e^K T^{+2} \\
&= 0
\end{aligned} \tag{32}$$

The fact that  $\partial_k F_\Sigma X^\Sigma - F_\Lambda \partial_k X^\Lambda = 0$  is a property of the special geometry of the moduli space [1].

Note that although the above derivation only requires  $z^i$  to be constant, it is not arbitrary. It is because in the case of  $\mathcal{F}^{-i} = 0$ , we have [4] that  $D_i Z = \frac{1}{2} \int \mathcal{F}^{+j} g_{ij} = 0 \Rightarrow \partial_i |Z| = 0$ , where  $Z$  is the central charge – a function of  $(p^\Lambda, q_\Lambda)$  and  $z$ , and so  $z$  is constrained to take the value for which when  $|Z|$  is at extremum.

Now, we are going to solve the Einstein equation (14) to obtain (20, 21).

The non-vanishing components of  $R_{\mu\nu}$  in the basis  $(dt, dr, r d\theta, r \sin \theta d\phi)$  are

$$R_{tt} = -e^{4U} \vec{\nabla}^2 U, \tag{33}$$

$$R_{rr} = 2U'^2 - \vec{\nabla}^2 U, \tag{34}$$

$$R_{\theta\theta} = -\vec{\nabla}^2 U, \tag{35}$$

$$R_{\phi\phi} = -\vec{\nabla}^2 U, \tag{36}$$

where  $U'$  is  $\frac{\partial}{\partial r} U$  and  $\vec{\nabla}^2$  is the spatial part of the Laplacian.

The energy momentum tensor for  $\mathcal{F}^\Lambda$  is

$$T_{\mu\nu} = -\frac{1}{2\pi} \text{Im} \mathcal{N}_{\Lambda\Sigma} (\mathcal{F}^\Lambda_{\mu\alpha} \mathcal{F}^{\Sigma\alpha}_\nu - \frac{1}{4} g_{\mu\nu} \mathcal{F}^\Lambda_{\alpha\beta} \mathcal{F}^{\Sigma\alpha\beta}). \tag{37}$$

To obtain  $T_{\mu\nu}$ , we first use (19) to calculate

$$\mathcal{F}^\Lambda_{t\alpha} \mathcal{F}^{\Sigma\alpha}_t = -\frac{Q^\Lambda Q^\Sigma}{r^4} e^{6U}, \tag{38}$$

$$\mathcal{F}^\Lambda_{r\alpha} \mathcal{F}^{\Sigma\alpha}_r = \frac{Q^\Lambda Q^\Sigma}{r^4} e^{2U}, \tag{39}$$

$$\mathcal{F}^\Lambda_{\theta\alpha}\mathcal{F}^{\Sigma\alpha}_\theta = -\frac{P^\Lambda P^\Sigma}{r^4}e^{2U} , \quad (40)$$

$$\mathcal{F}^\Lambda_{\phi\alpha}\mathcal{F}^{\Sigma\alpha}_\phi = -\frac{P^\Lambda P^\Sigma}{r^4}e^{2U} \quad (41)$$

$$\mathcal{F}^\Lambda_{\alpha\beta}\mathcal{F}^{\Sigma\alpha\beta} = 2\frac{e^{4U}}{r^4}(P^\Lambda P^\Sigma - Q^\Lambda Q^\Sigma) . \quad (42)$$

From above and (37), the non-zero components of  $T_{\mu\nu}$  are

$$T_{tt} = \frac{1}{4\pi}\text{Im}\mathcal{N}_{\Lambda\Sigma}\frac{e^{6U}}{r^4}(P^\Lambda P^\Sigma + Q^\Lambda Q^\Sigma) , \quad (43)$$

$$T_{rr} = -\frac{1}{4\pi}\text{Im}\mathcal{N}_{\Lambda\Sigma}\frac{e^{2U}}{r^4}(P^\Lambda P^\Sigma + Q^\Lambda Q^\Sigma) , \quad (44)$$

$$T_{\theta\theta} = \frac{1}{4\pi}\text{Im}\mathcal{N}_{\Lambda\Sigma}\frac{e^{2U}}{r^4}(P^\Lambda P^\Sigma + Q^\Lambda Q^\Sigma) , \quad (45)$$

$$T_{\phi\phi} = \frac{1}{4\pi}\text{Im}\mathcal{N}_{\Lambda\Sigma}\frac{e^{2U}}{r^4}(P^\Lambda P^\Sigma + Q^\Lambda Q^\Sigma) . \quad (46)$$

Hence, the Einstein-Maxwell equation, (14),

$$R_{tt} - 8\pi T_{tt} = 0 , \quad (47)$$

$$R_{rr} - 8\pi T_{rr} = 0 , \quad (48)$$

$$R_{\theta\theta} - 8\pi T_{\theta\theta} = 0 , \quad (49)$$

$$R_{\phi\phi} - 8\pi T_{\phi\phi} = 0 , \quad (50)$$

gives, respectively,

$$\vec{\nabla}^2 U + 2\text{Im}\mathcal{N}_{\Lambda\Sigma}\frac{e^{2U}}{r^4}(P^\Lambda P^\Sigma + Q^\Lambda Q^\Sigma) = 0 , \quad (51)$$

$$2U'^2 - \vec{\nabla}^2 U + 2\text{Im}\mathcal{N}_{\Lambda\Sigma}\frac{e^{2U}}{r^4}(P^\Lambda P^\Sigma + Q^\Lambda Q^\Sigma) = 0 , \quad (52)$$

$$\vec{\nabla}^2 U + 2\text{Im}\mathcal{N}_{\Lambda\Sigma}\frac{e^{2U}}{r^4}(P^\Lambda P^\Sigma + Q^\Lambda Q^\Sigma) = 0 , \quad (53)$$

$$\vec{\nabla}^2 U + 2\text{Im}\mathcal{N}_{\Lambda\Sigma}\frac{e^{2U}}{r^4}(P^\Lambda P^\Sigma + Q^\Lambda Q^\Sigma) = 0 . \quad (54)$$

This amounts to

$$\vec{\nabla}^2 U = -2\text{Im}\mathcal{N}_{\Lambda\Sigma}\frac{e^{2U}}{r^4}(P^\Lambda P^\Sigma + Q^\Lambda Q^\Sigma) , \quad (55)$$

and

$$\overline{\nabla}^2 e^{-U} = 0 , \quad (56)$$

since (56) is equivalent to  $\overline{\nabla}^2 U - U'^2 = 0$ .

Therefore, from (55, 56),  $e^{-U}$  is harmonic and has solution

$$e^{-U} = 1 + \frac{M}{r} , \quad (57)$$

$$M^2 = -2 \operatorname{Im} \mathcal{N}_{\Lambda\Sigma} (Q^\Lambda Q^\Sigma + P^\Lambda P^\Sigma) , \quad (58)$$

which was necessary to prove, as suggested in eqs. (20, 21).

When there are no vector multiplets,  $\operatorname{Im} \mathcal{N} = -\frac{1}{2}$  by (8) and  $M^2$  reduces to  $P^2 + Q^2$ , the well known Reissner-Nordstrom solution.

Now we will show that the mass of the black hole is actually equal to the extremal value of the central charge. For this purpose we have to rewrite our mass formula which follows from the solution of field equations and is given in terms of  $P$  and  $Q$  charges of  $\mathcal{F}$ 's to the one which employs the symplectic covariant charges  $(p, q)$ , defined in eq. (15). In terms of the charges of  $\mathcal{F}^\Lambda$ ,

$$\begin{pmatrix} p^\Lambda \\ q_\Lambda \end{pmatrix} = \begin{pmatrix} 2P^\Lambda \\ \operatorname{Re} \mathcal{N}_{\Lambda\Sigma} 2P^\Sigma - \operatorname{Im} \mathcal{N}_{\Lambda\Sigma} 2Q^\Sigma \end{pmatrix} \quad (59)$$

with the inverse

$$\begin{pmatrix} P^\Lambda \\ Q^\Lambda \end{pmatrix} = \frac{1}{2} \begin{pmatrix} p^\Lambda \\ (\operatorname{Im} \mathcal{N}^{-1} \operatorname{Re} \mathcal{N} p)^\Lambda - (\operatorname{Im} \mathcal{N}^{-1} q)^\Lambda \end{pmatrix}. \quad (60)$$

Thus using (58) we get

$$\begin{aligned} M^2 &= -2 \operatorname{Im} \mathcal{N}_{\Lambda\Sigma} (Q^\Lambda Q^\Sigma + P^\Lambda P^\Sigma) \\ &= -\frac{1}{2} (p^\Lambda, q_\Lambda) \begin{pmatrix} (\operatorname{Im} \mathcal{N} + \operatorname{Re} \mathcal{N} \operatorname{Im} \mathcal{N}^{-1} \operatorname{Re} \mathcal{N})_{\Lambda\Sigma} & (-\operatorname{Re} \mathcal{N} \operatorname{Im} \mathcal{N}^{-1})_\Lambda^\Sigma \\ (-\operatorname{Im} \mathcal{N}^{-1} \operatorname{Re} \mathcal{N})_\Sigma^\Lambda & (\operatorname{Im} \mathcal{N}^{-1})^{\Lambda\Sigma} \end{pmatrix} \begin{pmatrix} p^\Sigma \\ q_\Sigma \end{pmatrix} \\ &= |Z|^2 + |Z_i|^2 , \end{aligned} \quad (61)$$

where  $Z$  is the central charge and  $Z_i$  is given by  $-\frac{1}{2} \int \mathcal{F}^{+\bar{j}} g_{i\bar{j}} = 0$  as  $\mathcal{F}^{-i} = 0$  [1]. Hence, the mass of the double-extreme black hole is equal to the extremal value of the central charge.

$$M = |Z|_{Z_i=0} . \quad (62)$$

This is in accordance with what is found in [4], in particular, (48) of [4] give the mass of a pure magnetic black hole ( $P \neq 0, Q = 0$ ) as  $-\frac{1}{2} p^\Lambda \text{Im} \mathcal{F}_{\Lambda\Sigma} p^\Sigma$ , and  $\text{Im} \mathcal{F}_{\Lambda\Sigma}$  here can be replaced by  $\text{Im} \mathcal{N}_{\Lambda\Sigma}$ .

Thus the double-extreme black holes have the extremal ADM mass for a given set of  $(p, q)$  charges when there is a relation between the charges and vector multiplet moduli, given in eq. (3).

$$p^\Lambda = i(\bar{Z} L^\Lambda - Z \bar{L}^\Lambda) , \quad q_\Lambda = i(\bar{Z} M_\Lambda - Z \bar{M}_\Lambda) . \quad (63)$$

This relation was derived in [4] from the constraint  $D_i Z = Z_i = \mathcal{F}^{-i} = 0$  and as shown here is required for the solution of the equations of motion and the consistency of the total picture.

In the most general case when we have arbitrary sections  $(L, M)$  this is the best form of a constraint between the charges and moduli, which is available. Note that the central charge is a linear functions of  $(p^\Lambda, q_\Lambda)$  charges and depends on moduli as shown in eq. (9). In principle eqs. (63) can be solved for  $z^i$  in terms of  $(p^\Lambda, q_\Lambda)$ . In what follows we will present few examples when this constraint can be solved so that the explicit form of the complex moduli  $z^i$  in terms of charges  $(p^\Lambda, q_\Lambda)$  can be found.

### III. AXION-DILATON DOUBLE-EXTREME BLACK HOLES

Up to this point all the results we derived hold for a general prepotential  $F$  or for a general symplectic sections, when the prepotential is not available. In this section we consider a special case of N=2 supergravity interacting with one vector multiplet. The prepotential is given by  $F = -iX^0 X^1$ , which is equivalent to  $N = 4$  supergravity theory with two vector

fields. We will express the moduli  $z = \frac{X^1}{X^0}$  and the central charge  $|Z|$  in terms of charges  $p^\Lambda$  and  $q_\Lambda$  where  $\Lambda = 0, 1$  and relate them to the corresponding moduli in N=4 theory.

With this prepotential, choosing the gauge  $X^0 = 1$

$$L^\Lambda = e^{K/2} X^\Lambda = e^{K/2} \begin{pmatrix} 1 \\ z \end{pmatrix}, \quad (64)$$

$$M_\Lambda = e^{K/2} F_\Lambda = -ie^{K/2} \begin{pmatrix} z \\ 1 \end{pmatrix}, \quad (65)$$

where

$$e^K = \frac{1}{2(z + \bar{z})}. \quad (66)$$

As mentioned in the previous Section,  $z$  is constrained according to eqs. (63) which forces  $z$  to depend on the charges. Eliminating  $\bar{Z}$  we can rewrite equations (63) as

$$L^\Lambda q_\Sigma - p^\Lambda M_\Sigma = iZ(\bar{L}^\Lambda M_\Sigma - L^\Lambda \bar{M}_\Sigma), \quad (67)$$

or in matrix form and using (64, 65),

$$e^{K/2} \begin{bmatrix} q_0 + ip^0 z & q_1 + ip^0 \\ q_0 z + ip^1 z & q_1 z + ip^1 \end{bmatrix} = -Ze^K \begin{bmatrix} z + \bar{z} & 2 \\ 2z\bar{z} & z + \bar{z} \end{bmatrix}. \quad (68)$$

From equations of the two diagonal components we can solve for  $z$  in terms of charges:

$$z = \frac{q_0 - ip^1}{q_1 - ip^0}. \quad (69)$$

This is consistent with equations from other components. This is the expression for  $z$  in terms of the charges. In order to keep  $e^K$  positive according to (66) one is further required to take  $\text{Re} z$  to be equal to the absolute value of the real part of eq. (69). i.e. to  $|q_0 q_1 + p^0 p^1|$ .

Now let us express the central charge in terms of  $p^\Lambda$  and  $q_\Lambda$  only. In terms of  $z$ ,  $p^\Lambda$  and  $q_\Lambda$ , by (64, 65), the central charge is given by [1]

$$Z = L^\Lambda q_\Lambda - M_\Lambda p^\Lambda \quad (70)$$

$$= e^{K/2} [(q_0 + ip^1) + (q_1 + ip^0)z]. \quad (71)$$

Substituting (69) and (66) we get

$$Z = \left( \frac{q_0 q_1 + p^0 p^1}{q_1^2 + p^{02}} \right)^{1/2} (q_1 + i p^0) , \quad (72)$$

and the mass of the double-extreme black hole is a function of charges:

$$M^2 = |Z|^2 = |q_0 q_1 + p^0 p^1| . \quad (73)$$

Now consider the model related to this one by symplectic transformation [1]. It corresponds to N=2 reduction of the SU(4) formulation of pure N=4 supergravity.

$$\hat{X}^0 = X^0 , \quad \hat{F}_0 = F_0 , \quad \hat{X}^1 = -F_1 , \quad \hat{F}_1 = X^1 . \quad (74)$$

and for charges we have

$$\hat{p}^0 = p^0 , \quad \hat{q}_0 = q_0 , \quad \hat{p}^1 = -q_1 , \quad \hat{q}_1 = p^1 . \quad (75)$$

Our solution for moduli becomes

$$z = \frac{\hat{q}_1 + i \hat{q}_0}{\hat{p}^0 - i \hat{p}^1} , \quad (76)$$

and again it is required that  $\text{Re} z = |\hat{p}^0 \hat{q}_1 - \hat{q}_0 \hat{p}^1|$ . The double-extremal black hole mass in this version is given by:

$$M^2 = |Z|^2 = |\hat{p}^0 \hat{q}_1 - \hat{q}_0 \hat{p}^1| . \quad (77)$$

This coincides with the double-extreme axion-dilaton black holes in SU(4) version of N=4 supergravity [5], [6].

#### IV. N=2 HETEROTIC VACUA

In heterotic string vacua space-time supersymmetry comes from the right-moving sector of the string theory. In particular, the graviton, an antisymmetric tensor, the dilaton and 2 abelian fields together with fermions constitute the vector-tensor multiplet. On shell an antisymmetric tensor combines with the dilaton into a complex scalar, which belongs to an

N=2 vector multiplet. Other vector multiplets originate from the left-moving sector. The corresponding theory can be defined in terms of a prepotential <sup>1</sup>

$$F = \frac{1}{2} d_{ABC} t^A t^B t^C = S \eta_{ij} t^i t^j, \quad X^0 = 1, \quad (78)$$

where

$$t^1 = S, \quad d_{ABC} = \begin{pmatrix} d_{1jk} = \eta_{jk} \\ 0 \text{ otherwise} \end{pmatrix}, \quad A, B, C = 1, 2, \dots, n+1, \quad (79)$$

and

$$\eta_{ij} = \text{diag}(+, -, - \dots, -), \quad i, j = 2, \dots, n+1. \quad (80)$$

This prepotential corresponds to the product manifold  $\frac{SU(1,1)}{U(1)} \times \frac{SO(2,n)}{SO(2) \times SO(n)}$ . The  $\frac{SU(1,1)}{U(1)}$  coordinate is the axion-dilaton field  $S$ . The remaining  $n$  complex moduli  $t^i$  are special coordinates of the  $\frac{SO(2,n)}{SO(2) \times SO(n)}$  manifold. In particular, when  $n = 2$  we have

$$F = \frac{1}{2} d_{ABC} t^A t^B t^C = \frac{1}{2} S [(t^2)^2 - (t^3)^2]. \quad (81)$$

If we introduce the notation

$$t^2 \equiv \frac{1}{2}(T + U), \quad t^3 \equiv \frac{1}{2}(T - U), \quad (82)$$

the prepotential becomes

$$F = \frac{1}{2} STU. \quad (83)$$

This theory is defined by 3 complex moduli and 4 gauge groups and the corresponding manifold is  $\frac{SU(1,1)}{U(1)} \times \frac{SO(2,2)}{SO(2) \times SO(2)}$ .

We would like to find the values of moduli of the double-extreme black holes for this model with  $n = 2$  as well as for the general case of arbitrary  $n$ .

Our method is to use the version of this theory which is related to the one described above by a singular symplectic transformation [1]. This version does not have a prepotential

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<sup>1</sup>We are using here notation of [2].

and is defined in terms of a symplectic sections. To avoid complicated formulae we will not use hats to describe this version of the theory but the sections as well as charges should not be associated with the prepotential version above. Thus the starting point to describe the N=2 heterotic vacua is [1]

$$\begin{pmatrix} X^\Lambda \\ F_\Lambda \end{pmatrix} = \begin{pmatrix} X^\Lambda \\ S X_\Lambda \end{pmatrix}, \quad X^\Lambda X_\Lambda \equiv X \cdot X = 0, \quad \Lambda = 0, 1, \dots, n+1. \quad (84)$$

The metric  $\eta_{\Lambda\Sigma} = \text{diag}(+, +, -, \dots, -)$  is used for changing the position of the indices:  $X_\Lambda = \eta_{\Lambda\Sigma} X^\Sigma$ . Note that  $X^\Lambda$  are not independent and satisfy the constraint  $X \cdot X = 0$ . From sections one can derive  $K$  and  $\mathcal{N}$  as follows:

$$K = -\ln [i(S - \bar{S})X \cdot \bar{X}] , \quad (85)$$

$$\mathcal{N}_{\Lambda\Sigma} = (S - \bar{S}) \frac{X_\Lambda \bar{X}_\Sigma + \bar{X}_\Lambda X_\Sigma}{X \cdot \bar{X}} + \bar{S} \eta_{\Lambda\Sigma} . \quad (86)$$

The Kähler metric can be obtained from the second derivative of  $K$ :

$$g_{S\bar{S}} = \frac{\partial}{\partial S} \frac{\partial}{\partial \bar{S}} K = \frac{1}{(2\text{Im } S)^2} , \quad (87)$$

$$g_{X^\Lambda \bar{X}^\Sigma} = \frac{\partial}{\partial X^\Lambda} \frac{\partial}{\partial \bar{X}^\Sigma} K = \frac{1}{X \cdot \bar{X}} \left( \frac{X_\Sigma \bar{X}_\Lambda}{X \cdot \bar{X}} - \eta_{\Lambda\Sigma} \right) . \quad (88)$$

Then the non-gravitational part of the Lagrangian is given by

$$\mathcal{L} = \frac{1}{(2\text{Im } S)^2} \partial S \partial \bar{S} + \frac{1}{X \cdot \bar{X}} \left( \frac{X_\Sigma \bar{X}_\Lambda}{X \cdot \bar{X}} - \eta_{\Lambda\Sigma} \right) \partial X^\Lambda \partial \bar{X}^\Sigma \quad (89)$$

$$+ \text{Im } S \left[ 2 \left( \frac{X_\Lambda \bar{X}_\Sigma + \bar{X}_\Lambda X_\Sigma}{X \cdot \bar{X}} \right) - \eta_{\Lambda\Sigma} \right] \mathcal{F}^\Lambda \mathcal{F}^\Sigma + \text{Re } S \eta_{\Lambda\Sigma} \mathcal{F}^{\Lambda*} \mathcal{F}^\Sigma . \quad (90)$$

Using  $F_\Lambda = S X_\Lambda$  in the stabilization eqs. (63) and we can bring them to the following form

$$p^\Lambda = i\bar{Z} e^{K/2} X^\Lambda - iZ e^{K/2} \bar{X}^\Lambda , \quad (91)$$

$$q_\Lambda = i\bar{Z} e^{K/2} S X_\Lambda - iZ e^{K/2} \bar{S} \bar{X}_\Lambda . \quad (92)$$

We can contract these equations with  $X$  using the constraint  $X \cdot X = 0$  and we get



$$X \cdot p = -iZe^{K/2} X \cdot \overline{X} , \quad (93)$$

$$X \cdot q = -iZe^{K/2} \overline{S} X \cdot \overline{X} . \quad (94)$$

These two equations imply

$$X \cdot q = \overline{S} X \cdot p . \quad (95)$$

The above equation is useful as all  $X \cdot q$  and  $\overline{X} \cdot q$  can be expressed in terms of  $\overline{S} X \cdot p$  and  $S \overline{X} \cdot p$  respectively. We contract equations (91) and (92) with  $p$  and  $q$  and get the following:

$$p^2 = i\overline{Z}e^{K/2} X \cdot p - iZe^{K/2} \overline{X} \cdot p \quad (96)$$

$$q \cdot p = i\overline{Z}e^{K/2} X \cdot q - iZe^{K/2} \overline{X} \cdot q \quad (97)$$

$$p \cdot q = i\overline{Z}e^{K/2} S X \cdot p - iZe^{K/2} \overline{S} \overline{X} \cdot p \quad (98)$$

$$q^2 = i\overline{Z}e^{K/2} S X \cdot q - iZe^{K/2} \overline{S} \overline{X} \cdot q \quad (99)$$

Using (95) to get rid of  $q$  on the right hand sides we get:

$$p^2 = i\overline{Z}e^{K/2} X \cdot p - iZe^{K/2} \overline{X} \cdot p \quad (100)$$

$$q \cdot p = i\overline{Z}e^{K/2} \overline{S} X \cdot p - iZe^{K/2} S \overline{X} \cdot p \quad (101)$$

$$p \cdot q = i\overline{Z}e^{K/2} S X \cdot p - iZe^{K/2} \overline{S} \overline{X} \cdot p \quad (102)$$

$$q^2 = i\overline{Z}e^{K/2} S \overline{S} X \cdot p - iZe^{K/2} S \overline{S} \overline{X} \cdot p \quad (103)$$

Now, comparing eq. (100) with eq. (103) we see that it can be satisfied only if

$$S \overline{S} = \frac{q^2}{p^2} . \quad (104)$$

The sum of eqs. (101) and (102) can be compared with eq. (100) from which we learn that the real part of the S-moduli is already defined in terms of charges.

$$S + \overline{S} = \frac{2p \cdot q}{p^2} . \quad (105)$$

From the above two equations we can obtain the fixed value of the axion-dilaton, which is the moduli on  $\frac{SU(1,1)}{U(1)}$  manifold:

$$S = \frac{p \cdot q}{p^2} + i \frac{(p^2 q^2 - (p \cdot q)^2)^{1/2}}{p^2}, \quad (106)$$

here the sign of  $\text{Im } S$  is chosen to be negative.

To calculate the central charge  $|Z|$  one can multiply eq. (100) on eq. (103) and subtract the product of eqs. (102) and (101).

$$p^2 q^2 - (p \cdot q)^2 = |Z|^2 e^K (2S\bar{S} - \bar{S}^2 - S^2) X \cdot p \bar{X} \cdot p \quad (107)$$

$$= |Z|^2 e^K (S - \bar{S})^2 |Z|^2 e^K (X \cdot \bar{X})^2 \quad (108)$$

$$= |Z|^4, \quad (109)$$

which leads to

$$|Z|^2 = (p^2 q^2 - (p \cdot q)^2)^{1/2}, \quad (110)$$

where we used (93) and the expression for  $e^K$ .

The next step is to find the moduli on  $\frac{SO(2,n)}{SO(2) \times SO(n)}$  manifold. For this purpose we multiply  $\bar{S}$  on eq. (91) and subtract eq. (92):

$$\bar{S} p^\Lambda - q^\Lambda = i \bar{Z} e^{K/2} (\bar{S} - S) X^\Lambda, \quad (111)$$

which leads to a beautiful equation:

$$\frac{X^\Lambda}{X^\Sigma} = \frac{\bar{S} p^\Lambda - q^\Lambda}{\bar{S} p^\Sigma - q^\Sigma}. \quad (112)$$

Here  $\bar{S}$  is given by

$$\bar{S} = \frac{p \cdot q}{p^2} + i \frac{(p^2 q^2 - (p \cdot q)^2)^{1/2}}{p^2}. \quad (113)$$

Note that the ratio  $\frac{X^\Lambda}{X^\Sigma}$  does not give us yet the moduli since  $X \cdot X = 0$ . This constraint can be solved, in particular using Calabi-Vesentini coordinates which can be identified with the special coordinates of rigid special geometry. However for local special geometry a suitable set of unconstrained moduli can be taken in the following form [1,2]

$$X^0 = -\frac{1}{2}(1 - \eta_{ij} t^i t^j) , \quad (114)$$

$$X^{i-1} = t^i , \quad i, j = 2, \dots, n+1 , \quad (115)$$

$$X^{n+1} = \frac{1}{2}(1 + \eta_{ij} t^i t^j) . \quad (116)$$

This solution of the constraints has the property that  $X^{n+1} - X^0 = 1$ . The solution for moduli is

$$t^i = \frac{X^{i-1}}{X^{n+1} - X^0} = \frac{\bar{S}p^{i-1} - q^{i-1}}{\bar{S}(p^{n+1} - p^0) - (q^{n+1} - q^0)} . \quad (117)$$

Thus the double-extreme black hole of N=2 supergravity with  $\frac{SU(1,1)}{U(1)} \times \frac{SO(2,n)}{SO(2) \times SO(n)}$  symmetry has the following properties. The  $n+1$  complex vector multiplet moduli are functions of charges:

$$t^1 = S = \frac{p \cdot q}{p^2} - i \frac{(p^2 q^2 - (p \cdot q)^2)^{1/2}}{p^2} , \quad (118)$$

$$t^i = \frac{\bar{S}p^{i-1} - q^{i-1}}{\bar{S}(p^{n+1} - p^0) - (q^{n+1} - q^0)} , \quad i = 2, \dots, n+1 . \quad (119)$$

The mass (proportional to the area of the black hole horizon) is given by

$$M^2 = |Z|^2 = (p^2 q^2 - (p \cdot q)^2)^{1/2} , \quad (120)$$

and the scalars in the hypermultiplets can take arbitrary values not specified by vector field charges.

Note that the moduli could be also rewritten as functions of the charges of the version of the theory which has the prepotential. The explicit singular symplectic transformation which relate those 2 set of charges is available [1,2]. It seems however that the expressions for moduli are more complicated and will not be given here.

One can focus on a simple case of just 3 vector multiplets and  $\frac{SU(1,1)}{U(1)} \times \frac{SO(2,2)}{SO(2) \times SO(2)}$  model with the following charges:  $(p^0, q_0), (p^1, q_1), (p^2, q_2), (p^3, q_3)$  of the 4 gauge groups. The  $O(2,n)$  scalar products are

$$p^2 = (p^0)^2 + (p^1)^2 - (p^2)^2 - (p^3)^2 , \quad (121)$$

$$q^2 = (q_0)^2 + (q_1)^2 - (q_2)^2 - (q_3)^2 , \quad (122)$$

$$p \cdot q = (p^0 q_0) + (p^1 q_1) + (p^2 q_2) + (p^3 q_3) . \quad (123)$$

The frozen values of 3 complex moduli are

$$\begin{aligned}
S &= \frac{p \cdot q}{p^2} - i \frac{(p^2 q^2 - (p \cdot q)^2)^{1/2}}{p^2} , \\
T &= \frac{\overline{S}(p^1 + p^2) - (q^1 + q^2)}{\overline{S}(p^3 - p^0) - (q^3 - q^0)} , \\
U &= \frac{\overline{S}(p^1 - p^2) - (q^1 - q^2)}{\overline{S}(p^3 - p^0) - (q^3 - q^0)} .
\end{aligned} \tag{124}$$

The mass of the generic extreme black holes in this class depends on charges and moduli, it is minimal for the black holes with regular horizons and given in eq. (120) when moduli are taking the extremal values given in eqs. (124). This accomplishes our goal of finding the explicit form of the moduli as functions of charges for double-extreme black holes.

## V. DISCUSSION

In this paper we have introduced a new concept of double-extreme black holes. They are characterized by the regular horizon and unbroken supersymmetry and their ADM mass is minimal for a given set of charges. The important new feature of these black holes is that the moduli are frozen in space at the values which minimize the ADM mass. We have found all N=2 dyonic double-extreme black holes.

The fact that the central charge of the gravitational dyons has an extremum in moduli space at the fixed values of charges follows from unbroken supersymmetry which is doubled near the black hole horizon [4]. However, not so many examples of the known black holes are available to verify this theorem. The new double-extreme black holes found in this paper for the most general coupling of N=2 supergravity to vector multiplets exhibit a relation between charges and moduli given in eq. (3) and found before in [4]. For the most general set of symplectic sections defining a given theory this relation, which we have called stabilization equation, is difficult to solve. However we have solved it for the so-called tree level heterotic N=2 vacua with arbitrary number of vector multiplets described by some of the classical

Calabi-Yau moduli space, see eqs. (118,119). The new expressions for moduli in terms of electric and magnetic charges have been found for arbitrary number of complex moduli, in particular for the complex values of  $S, T, U$  moduli in  $\frac{SU(1,1)}{U(1)} \times \frac{SO(2,2)}{SO(2) \times SO(2)}$  model with 4 electric and 4 magnetic charges, see eq. (124). Those are the main new technical results of this work.

The fact that these new solutions have been found allows some room for speculation along the lines of “superpotential-black-hole-relation” [6]. In  $\frac{SU(1,1)}{U(1)} \times \frac{SO(2,n)}{SO(2) \times SO(n)}$  model we have found the frozen values of moduli by solving the equations of motion for the double-extreme black holes. Quite recently the spontaneous breaking of  $N=2$  into  $N=1$  supersymmetry was studied in this model where also the hypermultiplets living on a  $\frac{SO(4,m)}{SO(4) \times SO(m)}$  manifold have been gauged [7]. It would be interesting to understand the total issue of partial breaking of supersymmetry from various points of view.

In particular, according to [8] the hypermultiplet charges originate from the expectation values of the 2-, 4- and 10-forms of the ten-dimensional theory and are related to R-R charges of the background. When only one type of these charges is not vanishing, as studied in [8], this leads to new type II vacua of string theory with the potential without a stable minimum. Only the 10-form has an expectation value  $\langle E \rangle = \nu_0 \epsilon^{(10)}$ . The corresponding 4-dimensional supersymmetric black hole has only a magnetic charge in one of the gauge groups. Only one component of the hypermultiplet in the low-energy action acquires a charge due to the gauging:  $D_\mu a = \nabla_\mu a + \nu_0 \tilde{A}_\mu$ .

However, if more of the 10d forms are not vanishing, e.g. if the 2-form  $\langle G \rangle = \nu_2^i \omega_i^{(2)}$  and/or the 4-form  $\langle F \rangle = \nu_4^i \omega_i^{(4)} + \nu_6 \epsilon^{(4)}$  are not vanishing, this will lead to more charges of the hypermultiplets. The corresponding new vacua will have some stable minima at fixed values of moduli since the corresponding functions of moduli are related to the black hole mass as a function of moduli and black hole charges [6]. And these functions are known to have stable minima [4] provided the corresponding supersymmetric black holes have regular horizons.

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